A Mathematical Model of Two Layered Pulsatile Blood Flow through Stenosed Arteries

S. U. Siddiqui, S. R. Shah, Geeta

Department of Mathematics, Harcourt Butler Technological Institute, Kanpur-208002, India
Email: geeta05_hbti@rediffmail.com

Abstract

In the present paper a two fluid model for pulsatile blood flow through stenosed artery has been developed. The model consists of a core region of suspension of all the erythrocytes assumed to be a Bingham plastic fluid and a peripheral layer of plasma as Newtonian fluid. The analytic expressions for blood flow characteristics namely velocity, wall shear stress, flow rate, plug core radius and effective viscosity are obtained by using perturbation method. The effects of body acceleration and slip velocity have been discussed. We have shown the variation of flow variables with the different parameters and discussed with the help of graph. It is observed that the velocity and flow rate increases but effective viscosity decreases, due to a slip velocity. Body acceleration further enhances the velocity and flow rate.

Keywords: Slip velocity; Body acceleration; peripheral plasma layer; Bingham plastic.

Introduction

The behavior of blood flow is highly affected by arterial stenosis. Due to the presence of arterial stenosis normal blood flow is disturbed. Partially occlusion of blood vessels due to deposition of lipids, such as cholesterol commonly known as stenosis, is one of the most frequently occurring diseases in cardiovascular system of the human. The Newtonian behavior may be true in larger arteries, but blood being a suspension of cells in plasma exhibits a non-Newtonian behavior at the low shear rates in small arteries. Numerous investigators have cited hydrodynamics factors playing an important role in the formation of stenosis and hence the mathematical modeling of blood flow through a stenosed tube is very important. Many authors have dealt with this problem, treating blood as a Newtonian fluid and assuming the flow to be steady.

Several authors have studied the pulsatile nature of blood flow considering blood as a Newtonian fluid and non-Newtonian fluid also. They found that under normal conditions, blood flow in the human circulatory system depends upon the pumping action of heart. The heart pump produces the pressure gradient throughout the arterial and venous network. This pressure gradient consists of two components, one of which is constant or non fluctuating and other fluctuating or pulsatile. [10-12], have studied the pulsatile flow of non –Newtonian fluid in stenosed artery by considering blood as Casson fluid and Herschel –Bulkley fluid. Chaturani & Sumy [2] have investigated the effects of non-Newtonian nature of blood and pulsatality on flow through a stenosed tube.
Along with the pulsatality, Body acceleration is an important factor in the study of blood flow. It plays a very important role in the normal as well as diseased arterial system. Shahed [14] studied the pulsatile flow of blood through a stenosed porous

medium under periodic body acceleration. They present a model of blood flow in a partially occluded tube subjected to both the pulsatile pressure gradient due to the normal heart action and periodic body acceleration. Closed form expressions have been obtained for the axial velocity, flow rate, fluid acceleration and shear stress.
Sud and Sekhon [17] have studied the pulsatile flow of blood through rigid circular tube subject to body acceleration, treating blood as a Newtonian fluid. Under exceptional circumstances, however humans may also be subject to whole body acceleration (or vibration) For eg. while riding in a vehicle or while flying in an aircraft or spacecraft, man may unintentionally be subjected to accelerations. Though human body can adapt to changes but prolonged exposure to high level unintended external acceleration may cause serious or even fatal situations on account of disturbances in blood flow. Headache, abdominal pain, increase in pulse rate, loss of vision, venous pooling of blood in the extremities and hemorrhage in the face, neck, eye sockets, lungs and brain are some of the symptoms which result from prolonged accelerations. Various investigators [7, 19, 20] have analyzed the effect of body acceleration on pulsatile flow of casson fluid through mild stenosed artery. They studied the effect of pulsatality and body acceleration through stenosed artery treating blood as casson and H-B fluid.
Several authors carried out the role of slip velocity in the blood flow through stenosed arteries and suggested the presence of red blood cells occurring in the slip condition at the vessel wall. [5,8] have developed mathematical models for blood flow through stenosed arterial segment, by taking a velocity slip condition at the constricted wall. Pulsatile flow of blood through a catheterized artery in the presence of different geometry of stenosis with a velocity slip at the stenotic wall has been investigated by [1, 18].
In all the above mentioned studies, only single layered blood flow has been considered but some theoretical and experimental analysis were performed to study the two layered blood flow under different physiological conditions. Srivastava, Rastogi & Vishnoi [16] have considered the two layered suspension of blood flow through overlapping stenosis. [4, 9, 11] have considered two-phase arterial blood flow through a composite stenosis. Some authors also considered the two layered flow with the effect of magnetic field. They studied the two fluid nonlinear mathematical models for pulsatile blood flow through stenosed arteries. He studied the pulsatile flow of a two fluid model for blood through stenosed narrow arteries at the low shear rate, assuming the suspension of all the erythrocytes in the core region of blood vessel at a Casson fluid and plasma in the peripheral layer as a Newtonian flow.

In the present model we have considered the two layered blood flow through stenosed artery. With the above motivation an attempt has been made to study the effect of slip velocity and body acceleration on the different flow variables assuming the body fluid blood as a model with the suspension of all the erythrocytes in the core region as Bingham plastic and the peripheral as a Newtonian fluid. We have seen the effect of various parameters on the different flow variables and compared with the existing models.

Mathematical formulation

Consider an axially symmetric, laminar, pulsatile and fully developed flow of blood in the axial direction through circular tube with an axially symmetric mild stenosis. It is assumed that the body fluid blood is represented by a two fluid model with a central layer of suspension of all erythrocytes as a Bingham plastic fluid and a peripheral layer of plasma as a Newtonian fluid. The artery length is assumed to be large enough as compared to its radius so that the entrance and exit, special wall effects can be neglected. The geometry of the arterial stenosis is shown in Fig. 1. Since the stenosis present in the artery is considered to be mild, the radial transport of blood is negligible and thus we have neglected the radial velocity of the blood in this study. Hence in the present study, the flow of blood is considered to be unidirectional and is in the axial direction.

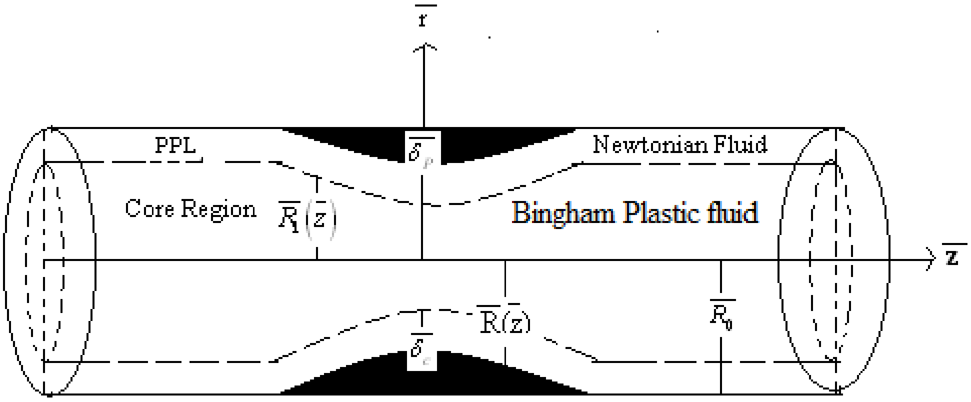

Figure 1. The geometry of an axially symmetric arterial stenosis

The geometry of stenosis in the peripheral region and core region are shown in Figure 1. & given by

$$\bar{R}(\bar{z}) = \begin{cases} \bar{R}_0 - \left(\dfrac{\bar{\delta}_P}{2}\right)\left\{1+\cos\left[\dfrac{2\pi}{\bar{L}_0}\left(\bar{z}-\bar{d}-\dfrac{\bar{L}_0}{2}\right)\right]\right\} & \text{in } \bar{d} \le \bar{z} \le \bar{d}+\bar{L}_0 \\ \bar{R}_0 & \text{in the normal artery region.} \end{cases} \quad (1)$$

$$\bar{R}_1(\bar{z}) = \begin{cases} \beta\bar{R}_0 - \dfrac{\bar{\delta}_B}{2}\left\{1+\cos\left[\dfrac{2\pi}{\bar{L}_0}\left(\bar{z}-\bar{d}-\dfrac{\bar{L}_0}{2}\right)\right]\right\} & \text{in } \bar{d} \le \bar{z} \le \bar{d}+\bar{L}_0 \\ \beta\bar{R}_0 & \text{in the normal artery region} \end{cases} \quad (2)$$

Where $\bar{R}(\bar{z})$ and $\bar{R}_1(\bar{z})$ are the radii of the with peripheral layer and core region respectively such that $\bar{R}_1(\bar{z}) = \beta\bar{R}(\bar{z})$, $\bar{R}_0$ and $\beta\bar{R}_0$ are the radii of the normal artery and core region of the normal artery respectively; $\bar{\delta}_P$ is the maximum height of the stenosis in the peripheral region, β is the ratio of the central core radius to the normal artery radius, $\bar{\delta}_B$ is the maximum height of the stenosis in the core region such that $\bar{\delta}_B = \beta\bar{\delta}_P$ and $\bar{z}_0$ is the half length of the stenosis. $\bar{L}_0$ is the length of stenosis; $\bar{d}$ indicates the location of the stenosis.

The law of conservation of mass for one dimensional fluid flow in the deformable tube gives the following equation of continuity in the core region and peripheral region.

$$\frac{\partial \overline{R_1}}{\partial \bar{t}} + \frac{\overline{R_1}}{2}\frac{\partial \overline{u_B}}{\partial \bar{z}} + \overline{u_B}\frac{\partial \overline{R_1}}{\partial \bar{z}} = 0 \text{ in } 0 \le \bar{r} \le \overline{R_1}(\bar{z}) \qquad (3)\backslash$$

$$\frac{\partial \overline{R}}{\partial \bar{t}} + \frac{\overline{R}}{2}\frac{\partial \overline{u_N}}{\partial \bar{z}} + \overline{u_N}\frac{\partial \overline{R}}{\partial \bar{z}} = 0 \text{ in } \overline{R_1}(\bar{z}) \le \bar{r} \le \overline{R}(\bar{z}) \qquad (4)$$

Where $\overline{u_B}$ the velocity of the Bingham plastic fluid in the core region and $\overline{u_N}$ is the velocity of the Newtonian fluid in the peripheral layer region. Since the blood flows through the narrow arteries at the low shear rates, therefore flow is slow and the viscous forces dominate over the inertial forces and thus, the magnitude of the convective terms are negligibly small. Therefore in the present model, we have neglected the convective terms in the momentum equation and body acceleration is taken as external force.

$$\overline{\rho_B}\frac{\partial \overline{u_B}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{z}} - \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\overline{\tau_B}\right) + \overline{F}(\bar{t}) \qquad 0 \le \bar{r} \le \overline{R_1}(\bar{z}) \qquad (5)$$

$$\overline{\rho_N}\frac{\partial \overline{u_N}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{z}} - \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\overline{\tau_N}\right) + \overline{F}(\bar{t}) \qquad \overline{R_1}(\bar{z}) \le \bar{r} \le \overline{R}(\bar{z}) \qquad (6)$$

$\overline{\tau_B}$ and $\overline{\tau_N}$ are the shear stresses for the casson fluid and Newtonian fluid respectively, $\overline{\rho_B}$ and $\overline{\rho_N}$ are the densities for cason fluid and Newtonian fluid respectively, $\overline{P}$ is the pressure and $\overline{F}(\bar{t})$ is the body acceleration.

The consecutive equation for Bingham Plastic fluid and Newtonian fluid are respectively given by

$$\begin{cases} \overline{\tau_B} = \overline{\tau_y} - \overline{\mu_B}\dfrac{\partial \overline{u_B}}{\partial \bar{r}} \text{ if } \overline{\tau_B} \ge \overline{\tau_y} \text{ and } \overline{R_p} \le \bar{r} \le \overline{R_1}(\bar{z}) \\ \dfrac{\partial \overline{u_B}}{\partial \bar{r}} = 0 \text{ if } \overline{\tau_B} \ge \overline{\tau_y} \text{ and } 0 \le \bar{r} \le \overline{R_p} \end{cases} \qquad (7)$$

$$\bar{\tau}_N = -\overline{\mu_N}\frac{\partial \overline{u_N}}{\partial \bar{r}} \text{ if } \overline{R_1}(\bar{z}) \le \bar{r} \le \overline{R}(\bar{z}) \qquad (8)$$

Where $\overline{\mu_B}$ and $\overline{\mu_N}$ are the viscosities of the Bingham plastic fluid and Newtonian fluid, respectively; $\overline{\tau_y}$ is the yield stress; $\overline{R_p}$ is the radius of the plug flow region.
The periodic body acceleration in the axial direction is given by

$$\overline{F}(\bar{t}) = a_0 \cos(\overline{\omega_b}\bar{t} + \phi) \qquad (9)$$

Where a_0 is its amplitude, $\overline{\omega_b} = 2\pi \overline{f_b}$, $\overline{f_b}$ is its frequency in Hz., ϕ is the lead angle of $\overline{F}(\overline{t})$ with respected to the heart action. The frequency of body acceleration $\overline{f_b}$ is assumed to be small so that wave effect can be neglected.

The pressure gradient at any $\bar{z}$ and $\bar{t}$ may be represented as follows.

$$\frac{\partial \overline{p}}{\partial \overline{z}}(\overline{z},\overline{t}) = A_0 + A_1 \cos(\overline{\omega_p t}) \tag{10}$$

Where A_0 the steady component of the pressure gradient is, A_1 is amplitude of the fluctuating component and $\overline{\omega_p} = 2\pi \overline{f_p}$ where $\overline{f_p}$ is the pulse frequency. Both A_0 and A_1 are function of $\bar{z}$.

we introduce the following non-dimensional variables.

$$\begin{gathered}
z = \frac{\overline{z}}{R_0},\ R(z) = \frac{\overline{R(z)}}{R_0},\ R_1(z) = \frac{\overline{R_1(z)}}{R_0},\ r = \frac{\overline{r}}{R_0},\ t = \overline{t w_p},\ w = \frac{\overline{w_b}}{w_p}, \\
\delta_p = \frac{\overline{\delta_p}}{R_0},\ \delta_c = \frac{\overline{\delta_c}}{R_0}, u_B = \frac{\overline{u_B}}{A_0 \overline{R_0}^2 / 4\overline{\mu_B}},\ u_N = \frac{\overline{u_N}}{A_0 \overline{R_0}^2 / 4\overline{\mu_N}}, \\
u_s = \frac{\overline{u_s}}{A_0 \overline{R_0}^2 / 4\overline{\mu_N}},\ \tau_B = \frac{\overline{\tau_B}}{A_0 \overline{R_0} / 2},\ \tau_N = \frac{\overline{\tau_N}}{A_0 \overline{R_0} / 2},\ e = \frac{A_1}{A_0}, \\
B = \frac{a_0}{A_0}, \theta = \frac{\overline{\tau_y}}{A_0 R_0 / 2}, \alpha_B^2 = \frac{\overline{R_0^2 \omega_p \rho_B}}{\overline{\mu_B}}, \alpha_N^2 = \frac{\overline{R_0^2 \omega_p \rho_N}}{\overline{\mu_N}}
\end{gathered} \tag{11}$$

Where α_B and α_N are pulsatile Reynolds's number for Bingham plastic fluid and Newtonian fluid respectively.

Using non-dimensional variables, equation (1) and (2) becomes

$$R(z) = \begin{cases} 1 - \left(\dfrac{\delta_P}{2}\right)\left\{1 + \cos\left[\dfrac{2\pi}{L_0}\left(z - d - \dfrac{L_0}{2}\right)\right]\right\} & \text{in } d \le z \le d + L_0 \\ 1 & \text{in the normal artery region.} \end{cases} \tag{12}$$

$$R_1(z) = \begin{cases} \beta - \dfrac{\delta_B}{2}\left\{1 + \cos\left[\dfrac{2\pi}{L_0}\left(z - d - \dfrac{L_0}{2}\right)\right]\right\} & \text{in } d \le z \le d + L_0 \\ \beta & \text{in the normal artery region} \end{cases} \tag{13}$$

The governing equation of motion given by equation (5) and (6) are simplified in the non dimensional form as

$$\alpha_B^2 \frac{\partial u_B}{\partial t} = 4 f(t) - \frac{2}{r}\frac{\partial}{\partial r}(r\tau_B) \qquad 0 \le r \le R_1(z) \tag{14}$$

$$\alpha_N^2 \frac{\partial u_N}{\partial t} = 4f(t) - \frac{2}{r}\frac{\partial}{\partial r}(r\tau_N) \qquad R_1(z) \le r \le R(z) \tag{15}$$

Where $f(t) = (1 + e\cos t) + B\cos(\omega t + \phi)$ (16)

The consecutive equation for Bingham Plastic fluid and Newtonian fluid are respectively given by equation (5) & (6) using non dimensional variables reduce to

$$\tau_B = \theta - \frac{1}{2}\frac{\partial u_B}{\partial r},\ if\ \tau_B \ge \theta,\ R_p \le r \le R_1(z) \tag{17}$$

$$\frac{\partial u_B}{\partial r} = 0, if\ \tau_B \le \theta,\ 0 \le r \le R_p \tag{18}$$

$$\tau_N = -\frac{1}{2}\frac{\partial u_N}{\partial r}, R_1(z) \le r \le R(z) \tag{19}$$

The boundary conditions are

$$\left.\begin{array}{l} \overline{\tau_B}\ \ is\ \ finite\ and\ \ \overline{\frac{\partial u_B}{\partial \bar{r}}} = 0\ at\ \bar{r} = 0 \\ \overline{u_N} = 0\ at\ r = \overline{R} \\ \overline{\tau_B} = \overline{\tau_N}\ \ and\ \ \overline{u_B} = \overline{u_N}\ \ at\ \bar{r} = \overline{R_1} \end{array}\right\} \tag{20}$$

The boundary conditions in the dimensionless form are

$$\left.\begin{array}{l} \tau_B\ is\ finite\ and\ \frac{\partial u_B}{\partial r} = 0\ at\ r = 0 \\ u_N = u_s\ at\ r = R \\ \tau_B = \tau_N\ and\ u_B = u_N\ at\ r = R_1 \end{array}\right\} \tag{21}$$

The non-dimensional volumetric flow rate is given by

$$Q = 4\int_0^{R(z)} ru(z,r,t)rdr \tag{22}$$

Where $Q(t) = \dfrac{\overline{Q}(\bar{t})}{\pi\left(\overline{R_0}\right)^4 A_0 / 8\overline{\mu_B}}$; $\overline{Q}(\bar{t})$ is the volumetric flow rate.

The effective viscosity $\overline{\mu_e}$ defined as

$$\overline{\mu_e} = \pi\left(-\frac{\partial \bar{p}}{\partial \bar{z}}\right)\left(\overline{R}\right)^4 / \overline{Q}\left(\bar{t}\right), \tag{23}$$

Can be expressed in the non dimensional form as

$$\mu_e = (R)^4 (1 + e\cos t) / Q(t) \tag{24}$$

3. Method of solution

Since it is not possible to find an exact solution to the system of nonlinear equations (14)-(19), the perturbation method is used to obtain the approximate solution to the unknowns u_B, u_N, τ_B and τ_N .when we non-dimensionalize the momentum equations (5) and (6) α_B^2 and α_N^2 occurs naturally and hence it is more appropriate to expand the equations (14)-(19) about α_B^2 and α_N^2.

Let us expand the plug core velocity u_p, the velocity in the core region u_B in the perturbation series of α_B^2 as below (where α_B^2 <<1)

$$u_p(z,t) = u_{0p}(z,t) + \alpha_B^2 u_{1p}(z,t) + ... \tag{25}$$

$$u_B(z,r,t) = u_{0B}(z,r,t) + \alpha_B^2 u_{1B}(z,r,t) + ... \tag{26}$$

$$R_p(z,t) = R_{0p}(z,t) + \alpha_B^2 R_{1p}(z,t) + ... \tag{27}$$

$$u_N(z,r,t) = u_{0N}(z,r,t) + \alpha_N^2 u_{1N}(z,r,t) + ... \tag{28}$$

Substituting the perturbation series expansion in equation (14), (17) and (18) and equating the power of α_B^2 and constant terms.

$$\frac{\partial}{\partial r}(r\tau_{0B}) = 2f(t)r, \frac{\partial u_{0B}}{\partial t} = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_{1B}), -\frac{\partial u_{0B}}{\partial r} = 2(\tau_{0B} - \theta), -\frac{\partial u_{1B}}{\partial r} = 2\tau_{1B} \tag{29}$$

Similarly, using the perturbation series expansion in equation (15) & (18) and equating the powers of α_N^2, the resulting equations of the peripheral region can be obtained as

$$\frac{\partial}{\partial r}(r\tau_{0N}) = 2f(t)r, \frac{\partial u_{0N}}{\partial t} = -\frac{2}{r}\frac{\partial}{\partial r}(r\tau_{1N}), -\frac{\partial u_{0N}}{\partial r} = 2(\tau_{0N}), -\frac{\partial u_{1N}}{\partial r} = 2\tau_{1N} \tag{30}$$

Using perturbation expansion in the given boundary condition and equating constant term and the terms containing α_N^2 & α_B^2 we get

$$\begin{aligned} &\tau_{0B} \text{ and } \tau_{1B} \text{ are finite at } r = 0, \\ &\tau_{0B} = \tau_{0N}, \tau_{1B} = \tau_{1N}, u_{0B} = u_{0N}, u_{1B} = u_{1N} \text{ at } r = R_1(z), \\ &u_{0N} = u_s, u_{1N} = 0 \text{ at } r = R(z) \end{aligned} \tag{31}$$

On solving equations (29) & (30) using equation (31) we can obtain the values for unknowns

$u_{0p}, u_{1p}, u_{0B}, u_{1B}, u_{0N}, u_{1N}, \tau_{0B}, \tau_{1B}, \tau_{0N}, \tau_{1N}$.

$$\tau_{0B} = f(t)r, \tau_{0N} = f(t)r \tag{32}$$

$$u_{0N} = u_s + f(t)\left(R^2 - r^2\right) \tag{33}$$

$$u_{0B} = u_s + f(t)\left(R^2 - r^2\right) - 2kf(t)\left(R_1 - r\right) \tag{34}$$

$$u_{0p} = u_s + f(t)\left(R^2 - R_{0p}^2\right) - 2kf(t)\left(R_1 - R_{0p}\right) \tag{35}$$

$$\tau_{1B} = -\frac{f'(t)}{24}\left[6R^2 r - 3r^3 - 4k\left(3R_1 r - 2r^2\right)\right] \tag{36}$$

$$\tau_{1N} = -\frac{f'(t)}{24}\left[6R^2 r - 3r^3 - 4k\frac{R_1^3}{r}\right] \tag{37}$$

$$u_{1N} = \frac{f'(t)}{48}\left[12R^2 r^2 - 3r^4 - 9R^4 - 16kR_1^3 \ln\left(\frac{r}{R}\right)\right] \tag{38}$$

$$u_{1B} = \frac{f'(t)}{48}\left[12R^2 r^2 - 3r^4 - 9R^4 - 4k\left\{6r_1 r^2 - \frac{8}{3}r^3 - \frac{10}{3}R_1^3 + R_1^3 \ln\left(\frac{R_1}{R}\right)\right\}\right] \tag{39}$$

$$u_{1p} = \frac{f'(t)}{48}\left[12R^2 R_{0p}^2 - 3R_{0p}^4 - 9R^4 - 4k\left\{6R_1 R_{0p}^2 - \frac{8}{3}R_{0p}^3 - \frac{10}{3}R_1^3 + R_1^3 \ln\left(\frac{R_1}{R}\right)\right\}\right] \tag{40}$$

Where $R = R(z), R_1 = R_1(z), k^2 = \dfrac{\theta}{f(t)}$. (41)

Neglecting the terms of $o(\alpha_B^2)$ and higher power of α_B in equation (27), the first approximation plug core radius can be obtained as

$$R_{0p} = \theta / f(t) = k^2$$

The expressions for axial velocities in the core and peripheral regions are obtained as

$$u_N = u_s + f(t)\left(R^2 - r^2\right) + \alpha_N^2 \frac{f'(t)}{48}\left[12R^2 r^2 - 3r^4 - 9R^4 - 16kR_1^3 \ln\left(\frac{r}{R}\right)\right] \tag{42}$$

$$u_B = u_s + f(t)\left(R^2 - r^2\right) + \alpha_B^2 \frac{f'(t)}{48}\left[\begin{array}{l} 12R^2 r^2 - 3r^4 - 9R^4 - 4k \\ \left\{6R_1 r^2 - \frac{8}{3}r^3 - \frac{10}{3}R_1^3 + R_1^3 \ln\left(\frac{R_1}{R}\right)\right\} \end{array}\right] \tag{43}$$

The expression for wall shear stress τ_w can be obtained as

$$\tau_w = \left(\tau_{0N} + \alpha_N^2 \tau_{1N}\right)_{r=R}$$

$$\tau_\omega = f(t)R - \frac{\alpha_N^2 f'(t)}{24}\left[3R^3 - 4k\frac{R_1^3}{R}\right] \tag{44}$$

From equation (22), (42) and (43) the volumetric flow rate is given by

$$Q = 4\int_0^{R_{0p}} r\left(u_{0p} + \alpha_B^2 u_{1p}\right)dr + 4\int_{R_{0p}}^{R_1} r\left(u_{0B} + \alpha_B^2 u_{1B}\right)dr + 4\int_{R_1}^{R} r\left(u_{0N} + \alpha_N^2 u_{1N}\right)dr$$

$$\begin{aligned} Q &= 2u_s R^2 + f(t)\left[R^4 - R_{0p}^4 - \frac{4k^2}{3}\left(R_1^3 - R_{0p}^3\right)\right] \\ &+ \frac{\alpha_B^2 f'(t)}{24}\left[\begin{array}{l} 6R^2R_{0p}^4 - 2R_{0p}^6 - R_1^6 + 6R^2R_1^4 - 9R^4R_1^2 \\ -4k\left\{3R_1R_{op}^4 - \frac{8}{5}R_{0p}^5 - \frac{7}{5}R_1^5 + R_1^5\ln\left(\frac{R_1}{R}\right)\right\} \end{array}\right] \\ &+ \frac{\alpha_N^2 f'(t)}{24}\left[9R^4R_1^2 - 6R^2R_1^4 - 4R^6 + R_1^6 + 8k\left\{2R_1^5\ln\left(\frac{R_1}{R}\right) + R^2R_1^3 - R_1^5\right\}\right] \end{aligned} \qquad (45)$$

The expression for effective viscosity μ_e can be obtained from equations (24) and (45)

$$\mu_e = (R)^4(1+e\cos t)\left[\begin{array}{l} 2u_s R^2 + f(t)\left[R^4 - R_{0p}^4 - \frac{4k^2}{3}\left(R_1^3 - R_{0p}^3\right)\right] \\ + \frac{\alpha_B^2 f'(t)}{24}\left[\begin{array}{l} 6R^2R_{0p}^4 - 2R_{0p}^6 - R_1^6 + 6R^2R_1^4 - 9R^4R_1^2 \\ -4k\left\{3R_1R_{op}^4 - \frac{8}{5}R_{0p}^5 - \frac{7}{5}R_1^5 + R_1^5\ln\left(\frac{R_1}{R}\right)\right\} \end{array}\right] \\ + \frac{\alpha_N^2 f'(t)}{24}\left[9R^4R_1^2 - 6R^2R_1^4 - 4R^6 + R_1^6 + 8k\left\{2R_1^5\ln\left(\frac{R_1}{R}\right) + R^2R_1^3 - R_1^5\right\}\right] \end{array}\right]^{-1}$$

(46)

The second approximation plug core radius R_{1p} can be obtained by neglecting terms of $o(\alpha_B^4)$ and higher power of α_B in equation (30) as

$$\begin{aligned} R_{1p} &= \frac{-\tau_{1B}\left(R_{0p}\right)}{f(t)} \\ &= \frac{f'(t)k^4}{24}\left(6R^2 - 5k - 12R_1\right) \end{aligned} \qquad (47)$$

From equation (27), (41) & (47), the expression for plug core radius can be obtained as

$$R_p = k - \frac{\alpha_B^2 f'(t)k^4}{24}\left(6R^2 - 5k - 12R_1\right) \qquad (48)$$

4. Result and Discussion

In order to estimate the effect of body acceleration and slip velocity by modeling blood as two layered flow. Perturbation techniques are used to solve the equations governing the fluid flow. The effect of pulsatality, stenosis, yield stress of the fluid, pressure gradient, shear stress, plug velocity, effective viscosity are investigated and discussed briefly with the help of graphs.

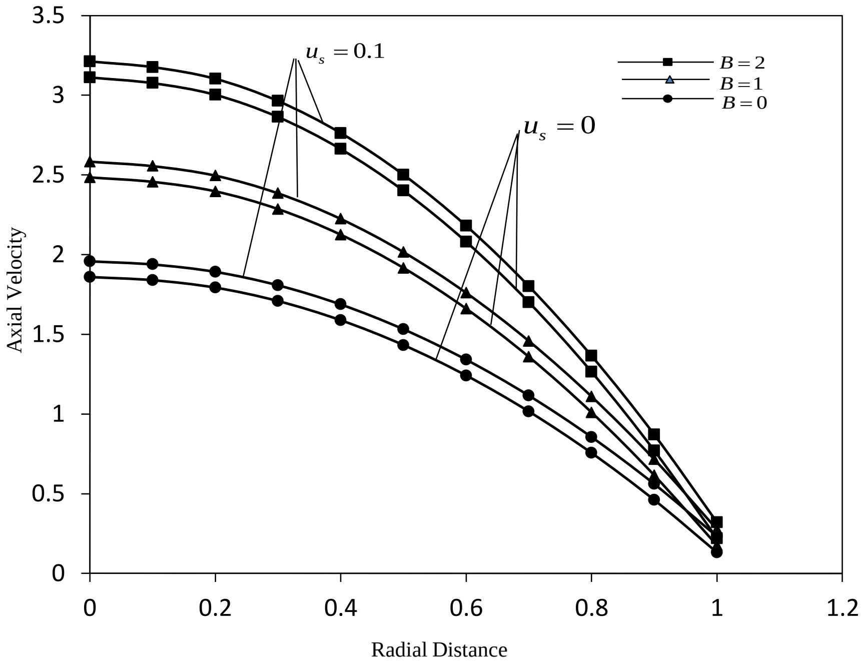

Figure 2. Velocity distribution for different values of B and u_s with $\alpha_N = 0.5, \phi = 0.2, \beta = 0.8, \delta_c = 0.2, \delta_p = 0.15, t = 45^{\circ}$

The velocity profile for different values of body acceleration parameter B and slip velocity u_s & for fixed values of peripheral stenosis height δ_p, pressure gradient e, time t wth radial distance r are shown in figure 2. The variation of axial velocity has been seen at the peak value of the stenosis i.e. at z=0.

It is observed that the maximum velocity is attained at r=0, after that it decreases gradually with the increase in the radius of the artery r. At the stenotic wall i.e. at $r = R(z)$ axial velocity is minimum for any value of body acceleration parameter B. However velocity is more when we consider the slip velocity at the wall than the no slip condition. Axial velocity is further increased with the increase in body acceleration.

Figure 3. Shows the variation of volumetric flow rate with the pressure gradient e for different values of yield stress θ and for fixed values of $L_0 = 1, \alpha_N = 0.5, \phi = 0.2, \beta = 0.8, z = 0$. We have seen the effects of both Body acceleration and slip velocity on the volumetric flow rate. It is found that the flow rate increases with the increase in pressure gradient parameter e for any value of θ, B. However it can be easily seen from the figure that flow rate is highly affected by the body acceleration and slip velocity. Flow rate is increases with the increase in body acceleration as well as with the increase in slip velocity. It is observed from the figure that the magnitude of flow rate in the presence of yield stress (i. e. $\theta = 0.1$) is less than it's magnitude in the absence of yield stress (i.e. $\theta = 0$).

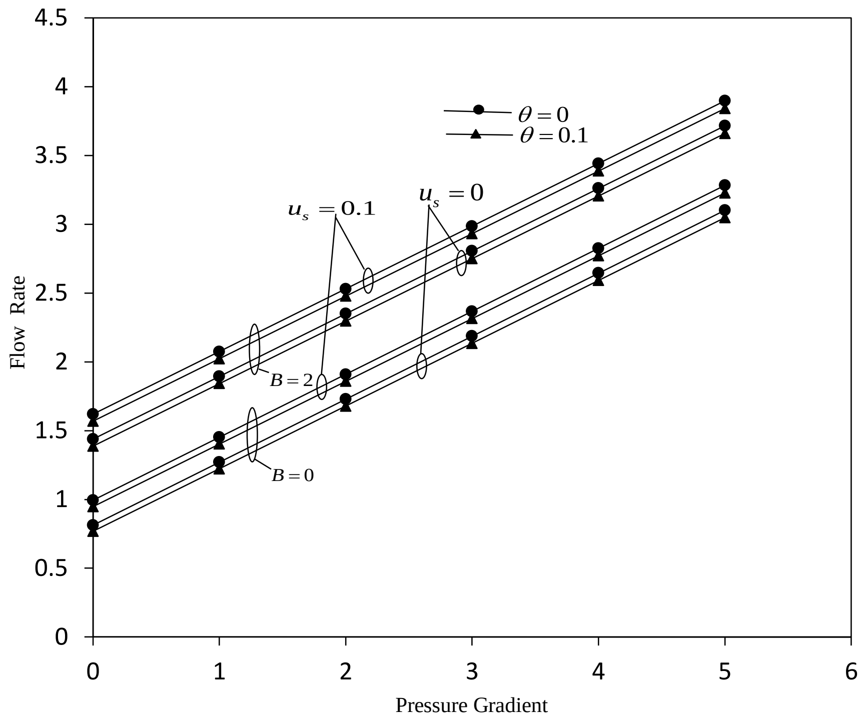

Figure 3. Variation of flow rate with the pressure gradient e for
$\alpha_B = \alpha_N = 0.5, R_{0p} = 0.2, \delta_c = 0.4$

Figure 4. depicts the variation of effective viscosity with the peripheral stenosis height for different values of body acceleration and yield stress θ.It is observed that effective viscosity decreases with the increase in body acceleration.It is also found that effective viscosity μ_e increases with the peripheral stenosis height for both the cases of no-slip and slip at wall. However increase in the value of yield stress θ increases the value of effective viscosity.The effective viscosity reduces when we apply slip at the wall for both the cases in presence and absence of yield stress.
Figure 5. and Figure 6. Shows the variation of wall shear stress with time t and peripheral stenosis height δ_p .from the figure 6. It is observed that wall shear stress decreases with the time t for the range $0^o \le t \le 180^o$ and after that it increases for the range $180^o \le t \le 360^o$. Wall shear stress increases in magnitude with increase in peripheral stenosis height.

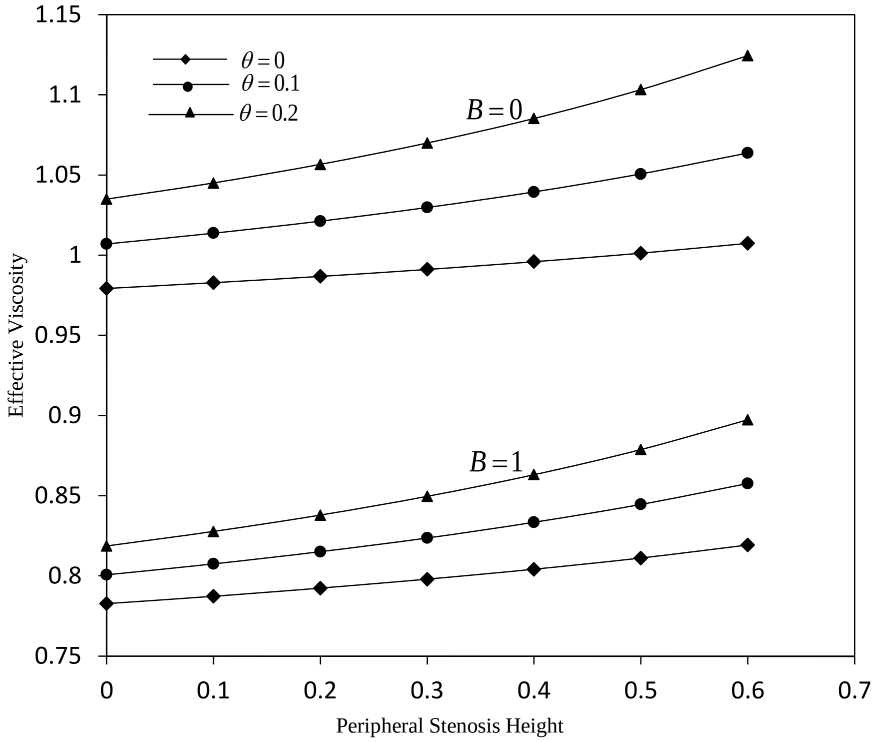

Figure 4. Variation of effective viscosity with the peripheral stenosis height for $\phi = 0.2, \delta_c = 0.5, u_s = 0, \omega = 1$

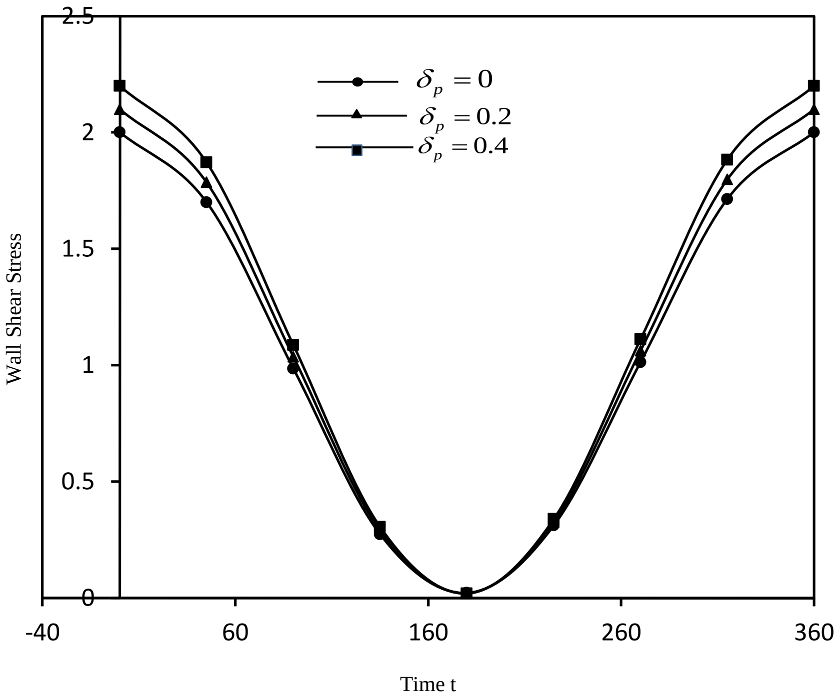

Figure 5. Variation of wall shear stress with the time t for $\alpha_N = 0.5, \theta = 0.1, \beta = 0.95$

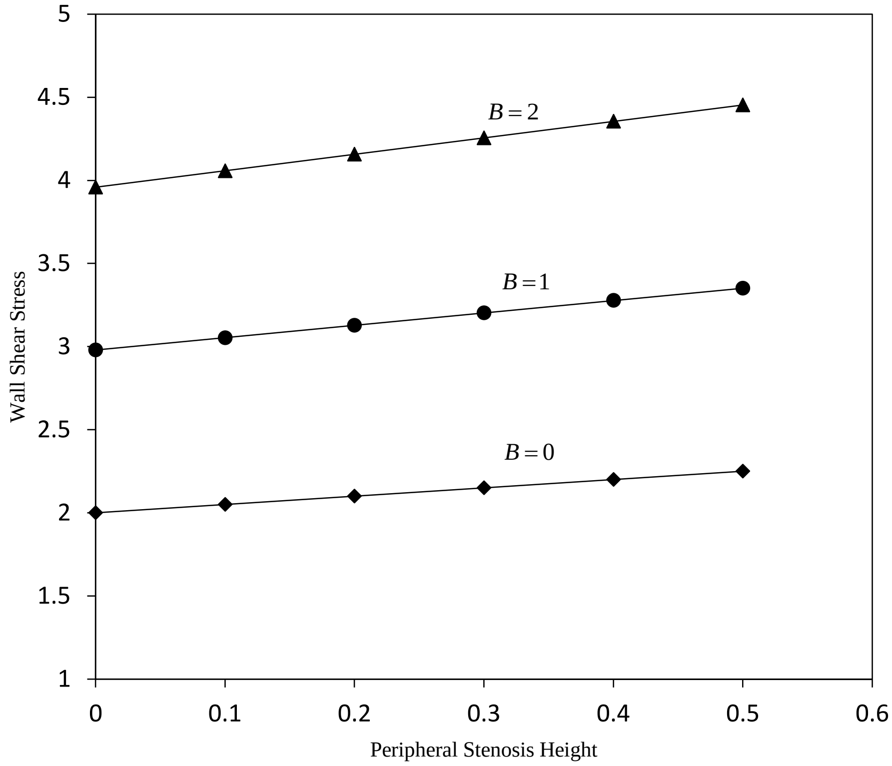

Figure 6. Variation of wall shear stress with peripheral stenosis height for $L_0 = 1, \phi = 0.2 \theta = 0.1, \delta_c = 0.5$

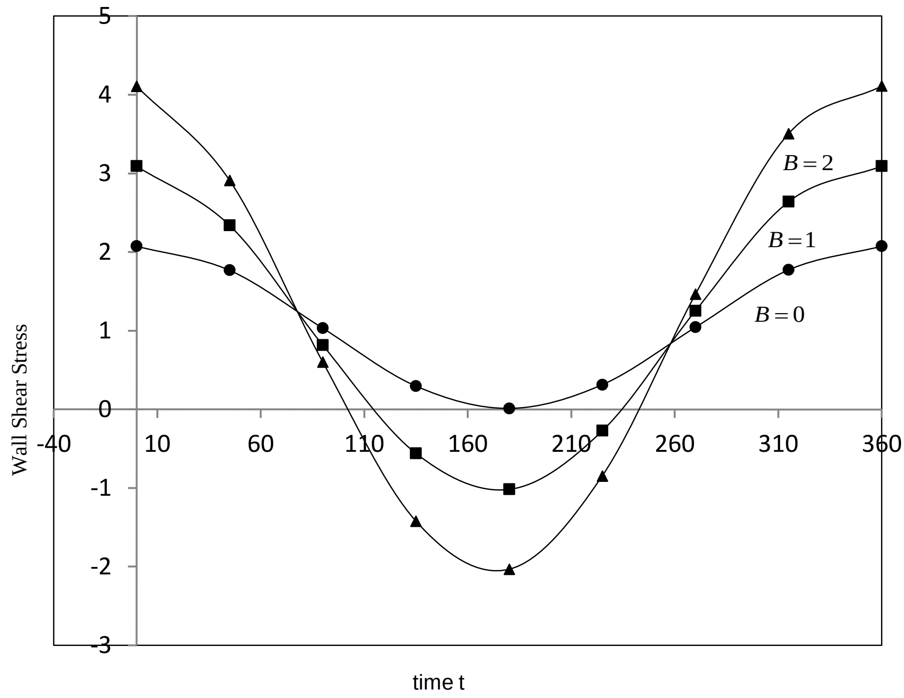

Figure 7. Variation of wall shear stress with the time for $L_0 = 1, \alpha_N = 0.5, \phi = 0.2, \beta = 0.8, e = 1$

From Figure 6. It is found that wall shear stress is highly affected by the body acceleration; in the absence of body acceleration wall shear is low in comparison to the presence of body acceleration. It is depicted that as we increase the body acceleration wall shear stress increases.

Figure 7. Shows the variation of wall shear stress with time t. the variation has been shown for a full scale of time $t(0 \le t \le 360)$ in degrees. It is found that the body acceleration parameter B highly influencing the wall shear stress τ_w in a stenosed artery, as body acceleration increases wall shear stress decreases. However, τ_w attains it's minimum at $t = 180^o$ and the maximum at $t = 0^o, t = 360^o$

Conclusion

The present mathematical model brings out the many interesting results due to the two layered blood flow i. e. due to peripheral layer. Effective viscosity increases as peripheral stenosis size δ_p increases It is found that plug core radius, wall shear stress, pressure drop and flow rate increases as the yield stress θ or stenosis size δ_p increases while keeping all other parameters constant. It is depicted that axial velocity and flow rate increases with the wall slip whereas effective viscosity decreases due to the presence of slip velocity. Body acceleration also play a very important role in blood flow modeling. Velocity and flow rate increases but wall shear stress decreases with the body acceleration. However μ_e is lowered for both the cases with or without stenosis due to the slip velocity. The present study can be extended by considering other fluid. In future one can consider the magnetic effect also for further research.

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